

# New Form of the T-Duality Due to the Stability of a Compact Dimension

Davoud Kamani

Received: 21 November 2007 / Accepted: 14 August 2008 / Published online: 30 August 2008  
© Springer Science+Business Media, LLC 2008

**Abstract** We study behaviors of a compact dimension and the  $T$ -duality, in the presence of the wrapped closed bosonic strings. When the closed strings interact and form another system of strings, the radius of compactification increases. This modifies the  $T$ -duality, which we call it as  $T$ -duality-like. Some effects of the  $T$ -duality-like will be studied.

**Keywords** Compactification ·  $T$ -Duality

## 1 Introduction

Stability of the radius of compactification has been studied from the various points of view [1–6]. We look at it from the aspect of the winding numbers and momentum numbers of the wrapped closed strings.

From the other side,  $T$ -duality (see, e.g. [7, 8]) leads to the ambiguities in the geometry and topology at the string length scale  $l_s = \sqrt{\alpha'}$  [9]. Consider strings propagating in some background fields (e.g. metric). These background fields should satisfy the equations of motion. Then, it turns out that different backgrounds can lead to the same physics without any observable difference between them. Therefore, this question arises: “What is the background metric?” and hence the background geometry is ambiguous.

Intuitively, the ambiguities in the geometry arise from the extended nature of the string. Features in the geometry which are smaller than the length scale  $l_s$  cannot be detected by using a string probe whose characteristic size is  $l_s$ . The simplest and most widely known example of the ambiguity in the geometry is the equivalence between a circle with radius  $R$  and a circle with radius  $\alpha'/R$ . A slightly more peculiar example is the equivalence between a circle with radius  $R = 2\sqrt{\alpha'}$  and a  $Z_2$  quotient of a circle (a line segment) with  $R = \sqrt{\alpha'}$ . This example demonstrates that the topology also is ambiguous. Removing these ambiguities leads to the breakdown of the  $T$ -duality. Breakdown of the  $T$ -duality from the

---

D. Kamani (✉)  
Faculty of Physics, Amirkabir University of Technology (Tehran Polytechnic), P.O. Box: 15875-4413,  
Tehran, Iran  
e-mail: [kamani@aut.ac.ir](mailto:kamani@aut.ac.ir)

nonperturbative [10] and nonlocality [11] points of view has been studied. We also receive it from a different approach.

Some closed strings are wound around a given compact dimension on a circle. We assume there are “ $K$ ” closed strings on this direction with the nonzero winding numbers  $\{n_1, n_2, \dots, n_K\}$ . These closed strings interact and form another system of closed strings. At first, we verify the stability of the compact dimension. We find that when at least two strings (before interactions) have winding numbers with different signs, the compact dimension is unstable. Thus, after interactions, the compactification radius increases. Specially, if all the wrapped closed strings can be released, the compact dimension will decompact.

Then, we find the dual radius, i.e.  $\tilde{R}$ , is different from  $\alpha'/R$ . This modifies the  $T$ -duality. We call it as “ $T$ -duality-like”. Consequently,  $l_s$  is not minimum length, and the spectrum of the mass operator on a circle of radius  $R$  and the same spectrum on the  $T$ -dual-like circle, which has the radius  $\tilde{R}$  are different. In other words, whether a circle respects the usual  $T$ -duality depends on the momentum and winding numbers of the wrapped closed strings.

This paper is organized as follows. In Sect. 2, the length of a closed string, wrapped around the compact dimension, will be obtained. In Sect. 3, the stability of the compact dimension, through the interactions of the initial  $K$  closed strings, will be studied. In Sect. 4, the modification of the  $T$ -duality for more than one closed string will be introduced. Section 5 is devoted to the conclusions.

Our analysis of a compact dimension depends on the winding and momentum numbers. These numbers appear in the zero modes of the bosonic strings. Thus, we consider only the bosonic strings.

## 2 Closed String Length

For a given periodic coordinate  $X$  there is the identification

$$X \equiv X + 2\pi R, \tag{1}$$

where  $R$  is the radius of compactification. A closed string may now wind around this compact dimension

$$X(\sigma + 2\pi, \tau) = X(\sigma, \tau) + 2\pi n R, \quad n \in \mathbf{Z}. \tag{2}$$

The integer  $n$  is the winding number of the closed string. The momentum of the closed string around this dimension is quantized

$$p = \frac{m}{R}, \quad m \in \mathbf{Z}, \tag{3}$$

where the integer “ $m$ ” is the momentum number of the closed string.

For the next purposes, we obtain the length of a closed string with the winding number “ $n$ ”. The string coordinates  $X^\mu(\sigma, \tau)$  mean that the string has definite position and shape. Thus, we can obtain its length. Let  $X^{\bar{\mu}}$  refers to the non-compact space coordinates, i.e.  $\bar{\mu} \in \{1, 2, \dots, 24\}$ . The compact coordinate is  $X^{25} = X$ . Therefore, the string length is given by

$$\begin{aligned} L_s &= \int_{\text{on string}} \sqrt{(dX)^2 + dX^{\bar{\mu}} dX_{\bar{\mu}}} \\ &= \int_0^{2\pi} d\sigma \sqrt{(\partial_\sigma X)^2 + \partial_\sigma X^{\bar{\mu}} \partial_\sigma X_{\bar{\mu}}} \Big|_{\tau=\tau_0}. \end{aligned} \tag{4}$$

The constant worldsheet time  $\tau_0$  indicates that the integration is over the string.

A closed string coordinate  $X^\mu(\sigma, \tau)$  has the solution

$$X^\mu(\sigma, \tau) = x^\mu + 2\alpha' p^\mu \tau + 2L^\mu \sigma + \frac{i}{2} \ell \sum_{n \neq 0} \frac{1}{n} \left( \alpha_n^\mu e^{-2in(\tau-\sigma)} + \tilde{\alpha}_n^\mu e^{-2in(\tau+\sigma)} \right), \tag{5}$$

where  $\ell = \sqrt{2\alpha'}$ , and  $L^\mu$  is zero for the non-compact coordinates. For the compact coordinate  $X$ , it is  $L = nR$ . Define  $u^\mu(\sigma, \tau_0)$  as in the following

$$u^\mu(\sigma, \tau_0) = \sum_{n \neq 0} \left( -\alpha_n^\mu e^{-2in(\tau_0-\sigma)} + \tilde{\alpha}_n^\mu e^{-2in(\tau_0+\sigma)} \right). \tag{6}$$

For a closed string with nonzero winding number, the length can be written as

$$L_s = 2L \int_0^\pi d\sigma \left[ 1 + \frac{\ell}{L} u + \left( \frac{\ell}{2L} \right)^2 u^2 + \left( \frac{\ell}{2L} \right)^2 u^{\bar{\mu}} u_{\bar{\mu}} \right]^{1/2}. \tag{7}$$

If  $\frac{\ell}{L} \ll 1$ , then we obtain  $L_s \approx n(2\pi R)$ . Neglecting the order  $(\frac{\ell}{L})^2$  also leads to the previous result, i.e.,

$$L_s = n(2\pi R) + \mathcal{O}(\ell^2/L). \tag{8}$$

Therefore, up to the order  $\mathcal{O}(\ell^2/L)$ , this is compatible with (2) (e.g. see [12]). In fact, (2) describes a closed string winding  $n$  times around the compact direction. Thus, up to this order, the wrapped closed strings completely touch the circle. In other words, the length of each loop of a wrapped closed string is not greater than the circle circumference.

### 3 Stability of the Compactification Radius

Assume there are “ $K$ ” wrapped closed strings with the nonzero winding numbers  $\{n_1, n_2, \dots, n_K\}$ . Some of these numbers are positive and the others are negative. There are also some other closed strings with zero winding number. Since these strings do not affect the analysis of the system we shall not consider them.

By the usual splitting-joining process two closed strings with the winding numbers  $n_1$  and  $n_2$  can turn into a closed string with the winding number  $N = n_1 + n_2$ , where each of  $n_1$  and  $n_2$  may be positive or negative. In fact, the total winding number is always conserved.

We consider the following processes. a) Strings with winding numbers of different signs, through the splitting and joining, interact and form a set of long strings. Since the un-wrapped strings do not affect the compactification, we consider only the wrapped closed strings of these long strings. These long strings have winding numbers of the same sign. However, the corresponding system is not the final system. b) The long and short closed strings in this system interact and form the final system of closed strings with the length  $2\pi R'$  for each loop of them. In other words, the final length of each loop determines the new radius of compactification  $R'$ . When  $R' > R$  (or  $R' = R$ ) we say that the *initial* compactification is unstable (stable). After transition to the final system, the circumference of the string loops remains constant. Therefore, the final radius of compactification  $R'$  is stable.

String thermodynamics [13–17] imposes a back-reaction on the background manifold. This back-reaction gives a dynamics to the spacetime. The winding modes of the closed

strings have important rule in this back-reaction to control the decompactification of the circled dimensions. That is, the compact dimensions become larger and larger to touch the wrapped closed strings. Therefore, increasing the radius of compactification increases the energy of the wrapped closed strings, and hence these strings prevent the expansion.

For decompactifying the compact dimension  $X$ , all the closed strings with nonzero winding numbers, through the splitting and joining processes, must interact and produce other closed strings with zero winding number. This can occur when the total winding number of the initial system

$$N = \sum_{i=1}^K n_i, \tag{9}$$

vanishes. For the case  $N \neq 0$  the compact dimension is unstable and hence the radius of compactification will increase.

As an example, let the initial system be two closed strings with the winding numbers  $n_1 = +2$  and  $n_2 = -1$ . They transit into a single string with  $N = n_1 + n_2 = +1$ . Therefore, the initial radius  $R$  changes to  $R'$ . The total string length is  $2(2\pi R) + (2\pi R)$ . This length determines the radius  $R'$ . It is given by  $3(2\pi R) = (2\pi R')$  and hence  $R' = 3R$ . Similarly,  $n_1 = -2$  and  $n_2 = +1$  also lead to  $N = -1$ , and therefore,  $R' = 3R$ . For the case  $n_1 = +2$  and  $n_2 = +1$ , we find  $N = +3$ . Thus, we have  $3(2\pi R) = 3(2\pi R')$  and hence  $R' = R$ . In the same way, the case  $n_1 = -2$  and  $n_2 = -1$  give  $N = -3$  and  $R' = R$ . In the two last cases, the radius is fixed and the winding number has been increased. In other words, the two last cases have stable radii.

Now we study the case that  $K$  initial closed strings interact and transit to the final system, i.e.,  $K'$  (with  $1 \leq K' \leq N$ ) closed strings with nonzero winding numbers  $\{N_1, N_2, \dots, N_{K'}\}$  and some zero winding strings. When the transited system is in its final state these numbers have the same sign. Otherwise, more interactions must occur to remove the strings with opposite winding numbers.

The total length of the strings in this transition is conserved, i.e.,  $|n_1|(2\pi R) + |n_2|(2\pi R) + \dots + |n_K|(2\pi R) = |N_1(2\pi R') + N_2(2\pi R') + \dots + N_{K'}(2\pi R')|$ . The conservation of the winding numbers also gives  $\sum_{i=1}^K n_i = \sum_{l=1}^{K'} N_l$ . These imply the relation

$$|N|R' = R \sum_{i=1}^K |n_i|. \tag{10}$$

If  $N \rightarrow 0$  the radius  $R'$  goes to infinity such that  $|N|R'$  to be finite and nonzero, i.e. equal to  $R \sum_{i=1}^K |n_i|$ . However, when  $R' \rightarrow \infty$ , the dimension  $X$  is decompactified. This occurs when the total winding number  $N$  vanishes. In other words, when  $N = 0$ , the compact dimension is completely unstable. The equation (10) also implies that the ratio  $R'/R$  is a rational number.

For the next purposes it is useful to write (10) in the form

$$R' = \frac{\sum_{i=1}^K |n_i|}{|\sum_{i=1}^K n_i|} R. \tag{11}$$

Generally, there is the inequality  $\sum_{i=1}^K |n_i| \geq |\sum_{i=1}^K n_i|$ . Therefore, we have  $R' \geq R$ . That is, after transition the radius of compactification increases, or at most remains unchanged. When all of the initial strings have winding numbers with the same sign, there is no transition and hence  $R' = R$ . In other words, the compact dimension is stable.

The equation (11) is consistent with the argument of [13]. That is, in the first moment of the Universe, the wrapped strings constricted all of the circular dimensions. These strings were highly likely to collide. The collisions involved string/antistring pairs, which led to annihilations. The collisions continually lessened the constriction and allowed three dimensions to expand.

#### 4 T-Duality for $K \geq 2$

$T$ -duality is an intriguing property of string theory. In its simplest form,  $T$ -duality is a remarkable fact that observable properties (such as the spectrum of the mass operator) of a closed string compactified on a circle with the radius  $R$  cannot be distinguished from that of a closed string compactified on a circle with the dual radius  $\alpha'/R$  (where  $l_s = \sqrt{\alpha'}$  is the fundamental length scale of string theory). Effectively,  $l_s$  turns out to be the shortest possible spatial distance, because a shorter string can always be reinterpreted as a longer string in the dual theory.

In the  $T$ -duality there are the following exchanges

$$\begin{aligned} n &\longleftrightarrow m, \\ R &\longleftrightarrow \frac{\alpha'}{R}. \end{aligned} \tag{12}$$

Thus, the left and the right components of the closed string momentum

$$\begin{aligned} p_L &= \frac{1}{2} \left( \frac{m}{R} + \frac{nR}{\alpha'} \right), \\ p_R &= \frac{1}{2} \left( \frac{m}{R} - \frac{nR}{\alpha'} \right), \end{aligned} \tag{13}$$

transform to  $p_L \rightarrow p_L$  and  $p_R \rightarrow -p_R$ . In addition, for the compact dimension, there are the following transformations for the oscillators of the closed string coordinate

$$\begin{aligned} \tilde{\alpha}_n &\longrightarrow \tilde{\alpha}_n, \\ \alpha_n &\longrightarrow -\alpha_n. \end{aligned} \tag{14}$$

The closed string mass operator is

$$M^2 = \frac{m^2}{R^2} + \frac{n^2 R^2}{\alpha'^2} - \frac{4}{\alpha'} + \frac{2}{\alpha'} \sum_{n=1}^{\infty} [(\alpha_{-n} \alpha_n + \tilde{\alpha}_{-n} \tilde{\alpha}_n) + (\alpha_{-n}^{\bar{\mu}} \alpha_{n\bar{\mu}} + \tilde{\alpha}_{-n}^{\bar{\mu}} \tilde{\alpha}_{n\bar{\mu}})]. \tag{15}$$

Under the transformations (12) and (14) the mass operator is invariant. This implies that the geometries with radii  $R$  and  $\alpha'/R$  are equivalent.

##### 4.1 Inconsistency of the $T$ -Duality for $K \geq 2$

In fact, the  $T$ -duality is a symmetry of the string theory (see Sects. 2 and 4.1 of the Ref. [7]). This symmetry is realized by invariance of the partition function of the compactified string theory under the duality transformations. Thus, in the compacted string theory, each

equation has a  $T$ -dual version equation with the same feature (e.g. see Sects. 1 and 4 of the Ref. [7]). We demonstrate that for a system with more than one closed string, i.e.  $K \geq 2$ , the generality of this statement (existence of the  $T$ -dual version equation) is not true. For example, on the basis of this statement, the  $T$ -dual version of (11) is

$$\tilde{R}' = \frac{\sum_{i=1}^K |\tilde{n}_i|}{|\sum_{i=1}^K \tilde{n}_i|} \tilde{R}, \quad (16)$$

where  $\tilde{R}$  and  $\tilde{R}'$  are  $T$ -duals of the radii  $R$  and  $R'$ , respectively. We have  $\tilde{n}_i = m_i$  and also  $\tilde{m}_i = n_i$ . In addition, if we use  $\tilde{R} = \alpha'/R$  and  $\tilde{R}' = \alpha'/R'$ , the above equation reduces to

$$R' = \frac{|\sum_{i=1}^K m_i|}{\sum_{i=1}^K |m_i|} R. \quad (17)$$

This is not correct. To see this, comparing (17) and (11) gives

$$\frac{\sum_{i=1}^K |n_i|}{|\sum_{i=1}^K n_i|} = \frac{|\sum_{i=1}^K m_i|}{\sum_{i=1}^K |m_i|}. \quad (18)$$

When the integer numbers  $\{n_i\}$  and  $\{m_i\}$  are arbitrary, this equation does not hold. For example, for two closed strings with the winding numbers  $n_1 = +2$  and  $n_2 = -1$  this equation gives  $3(|m_1| + |m_2|) = |m_1 + m_2|$ . For any nonzero numbers  $m_1$  and  $m_2$  it does not hold.

For solving the problem we should change the  $T$ -duality transformation of the radius  $R$ . In other words, we shall see that the  $T$ -dual version of  $R$ , i.e.  $\tilde{R} = \alpha'/R$  holds only for the case  $K = 1$ . That is, for the systems with  $K \geq 2$  it is incorrect. The correct form of it depends on the winding numbers and momentum numbers of the closed strings in the initial system, see (21).

## 4.2 $T$ -Duality-Like Transformations

Since the final radius of the compacted dimension  $R'$  is stable, it obeys the usual  $T$ -duality exchange  $R' \longleftrightarrow \alpha'/R'$ . Due to the initial strings with winding numbers of different signs, the initial radius  $R$  is unstable. Therefore, it does not obey the usual  $T$ -duality exchange. According to these, we introduce the following transformations

$$\begin{aligned} n_i &\longleftrightarrow m_i, \\ R' &\longleftrightarrow \frac{\alpha'}{R'}, \\ R &\longrightarrow \tilde{R}. \end{aligned} \quad (19)$$

Using these transformations and dualizing (11), lead to the equation

$$\frac{\alpha'}{R'} = \frac{\sum_{i=1}^K |m_i|}{|\sum_{i=1}^K m_i|} \tilde{R}. \quad (20)$$

The equations (11) and (20) define the dual radius  $\tilde{R}$ ,

$$\tilde{R} = \Omega_K \frac{\alpha'}{R}, \quad (21)$$

where

$$\Omega_K = \frac{|\sum_{i=1}^K m_i| |\sum_{i=1}^K n_i|}{\sum_{i=1}^K |m_i| \sum_{i=1}^K |n_i|}. \tag{22}$$

Therefore, the dual radius of the initial system depends on all momentum and winding numbers of that system. Since the numerator of  $\Omega_K$  is less than or equal to its denominator, we have  $\Omega_K \leq 1$ . This gives the inequality

$$\tilde{R} \leq \frac{\alpha'}{R}. \tag{23}$$

If the initial system contains closed strings with momentum numbers of the same sign, and also winding numbers of the same sign, we obtain  $\Omega_K = 1$ . This system does not have transition, and hence the compact dimension  $X$  is stable. The dual picture of this system is given by the usual  $T$ -duality. If there is one closed string in the initial system, i.e.  $K = 1$ , again there is no transition. Thus, we have  $\Omega_1 = 1$  and hence  $\tilde{R} = \alpha'/R$ , as expected.

In the usual  $T$ -duality, it is believed that there is no radius smaller than the self-dual radius  $\tilde{R} = R = \sqrt{\alpha'}$ . This indicates the minimum distance scale. In our duality, the self-dual radius is  $\tilde{R} = R = \sqrt{\alpha'\Omega_K}$ . This depends on the momentum and winding numbers of the wrapped closed strings. Since  $\Omega_K \leq 1$ , the self-dual radius is less than  $\sqrt{\alpha'}$ . In other words, detecting the distances less than  $\sqrt{\alpha'}$ , by two or more appropriate wrapped closed strings is possible. In addition, detecting the distances less than  $\sqrt{\alpha'}$ , by one closed string is impossible.

The relation (21) is invariant under the exchanges  $n_i \leftrightarrow m_i$  and  $R \leftrightarrow \tilde{R}$ . Let us write it in the form  $R\tilde{R} = \alpha'\Omega_K$ . When  $N = \sum_{i=1}^K n_i$  goes to zero, we have  $\Omega_K \rightarrow 0$ , and hence  $\tilde{R} \rightarrow 0$ . To see this, for  $N \rightarrow 0$  (11) implies that  $R' \rightarrow \infty$ . Therefore, (20) gives  $\tilde{R} \rightarrow 0$ . Similarly, when  $\sum_{i=1}^K m_i$  vanishes, (20) leads to  $R' \rightarrow 0$ . Thus, from (11) we obtain  $R \rightarrow 0$ . However, if the radii  $\tilde{R}$  and  $R$  are nonzero, the initial system has nonzero total momentum number and total winding number.

According to (19), the left and the right components of the momentum, i.e. (13), transform as in the following

$$\begin{aligned} p_L &\longrightarrow \frac{1}{2} \left( \frac{1}{\Omega_K} \frac{nR}{\alpha'} + \Omega_K \frac{m}{R} \right), \\ p_R &\longrightarrow \frac{1}{2} \left( \frac{1}{\Omega_K} \frac{nR}{\alpha'} - \Omega_K \frac{m}{R} \right). \end{aligned} \tag{24}$$

These hold for each of the  $K$  closed strings. For simplicity we use  $n$  and  $m$  instead of  $n_i$  and  $m_i$ . In terms of the zero modes  $\alpha_0 = \ell p_R$  and  $\tilde{\alpha}_0 = \ell p_L$ , these transformations take the forms

$$\begin{aligned} \tilde{\alpha}_0 &\longrightarrow \frac{1}{2} \left[ \left( \Omega_K - \frac{1}{\Omega_K} \right) \alpha_0 + \left( \Omega_K + \frac{1}{\Omega_K} \right) \tilde{\alpha}_0 \right], \\ \alpha_0 &\longrightarrow -\frac{1}{2} \left[ \left( \Omega_K + \frac{1}{\Omega_K} \right) \alpha_0 + \left( \Omega_K - \frac{1}{\Omega_K} \right) \tilde{\alpha}_0 \right]. \end{aligned} \tag{25}$$

Similar to the usual  $T$ -duality, we generalize these transformations to the nonzero mode oscillators

$$\begin{aligned} \tilde{\alpha}_n &\longrightarrow \frac{1}{2} \left[ \left( \Omega_K - \frac{1}{\Omega_K} \right) \alpha_n + \left( \Omega_K + \frac{1}{\Omega_K} \right) \tilde{\alpha}_n \right], \\ \alpha_n &\longrightarrow -\frac{1}{2} \left[ \left( \Omega_K + \frac{1}{\Omega_K} \right) \alpha_n + \left( \Omega_K - \frac{1}{\Omega_K} \right) \tilde{\alpha}_n \right], \end{aligned} \tag{26}$$

where  $n$  is any integer number. For  $\Omega_K = 1$  (e.g.  $K = 1$ ), these reduce to the usual  $T$ -duality transformations (14), as expected.

#### 4.2.1 Transformations of the Compact Coordinate and the Mass Operator

The compact coordinate of a closed string  $X(\sigma, \tau)$  has been given by (5) without the index  $\mu$ . According to  $p = p_L + p_R$  and  $L = \alpha'(p_L - p_R)$ , from the transformations (24) and (26) we obtain

$$\begin{aligned} X \longrightarrow X' &= \frac{1}{\Omega_K} x' + \frac{2}{\Omega_K} nR\tau + 2\alpha' \Omega_K \frac{m}{R} \sigma \\ &+ \frac{i}{4} \ell \sum_{n \neq 0} \frac{1}{n} \left\{ \left[ \left( \Omega_K - \frac{1}{\Omega_K} \right) \alpha_n + \left( \Omega_K + \frac{1}{\Omega_K} \right) \tilde{\alpha}_n \right] e^{-2in(\tau+\sigma)} \right. \\ &\left. - \left[ \left( \Omega_K + \frac{1}{\Omega_K} \right) \alpha_n + \left( \Omega_K - \frac{1}{\Omega_K} \right) \tilde{\alpha}_n \right] e^{-2in(\tau-\sigma)} \right\}. \end{aligned} \tag{27}$$

Since the transformations (24) give  $p_L + p_R \leftrightarrow \frac{1}{\Omega_K}(p_L - p_R)$ , we also introduced the exchange

$$x \longleftrightarrow \frac{1}{\Omega_K} x', \tag{28}$$

where  $x = x_L + x_R$  and  $x' = x_L - x_R$ . Therefore, the dual coordinate of a string depends on the momentum and winding numbers of all closed strings, presented in the system. According to (5), we can write  $X = X_L + X_R$ . However, (27) implies  $X' \neq Y_L - Y_R$  where  $Y_L$  and  $Y_R$  are functions of  $\tau + \sigma$  and  $\tau - \sigma$ , respectively. Note that (24), (26) and (28) also lead to the transformation  $X' \rightarrow X$ , as expected.

The mass operator (15) under (19) and (26) transforms to

$$\begin{aligned} M^2 \longrightarrow M'^2 &= \left( \frac{1}{\Omega_K} \frac{nR}{\alpha'} \right)^2 + \left( \Omega_K \frac{m}{R} \right)^2 - \frac{4}{\alpha'} + \frac{1}{\alpha'} \sum_{n=1}^{\infty} \left[ \left( \Omega_K^2 + \frac{1}{\Omega_K^2} \right) (\alpha_{-n} \alpha_n + \tilde{\alpha}_{-n} \tilde{\alpha}_n) \right. \\ &\left. + \left( \Omega_K^2 - \frac{1}{\Omega_K^2} \right) (\alpha_{-n} \tilde{\alpha}_n + \tilde{\alpha}_{-n} \alpha_n) \right] + \frac{2}{\alpha'} \sum_{n=1}^{\infty} (\alpha_{-n}^{\tilde{\mu}} \alpha_{n\tilde{\mu}} + \tilde{\alpha}_{-n}^{\tilde{\mu}} \tilde{\alpha}_{n\tilde{\mu}}). \end{aligned} \tag{29}$$

The mass operator  $M^2$  can be written as  $M^2 = M_L^2 + M_R^2$ , while due to the cross terms, the mass operator  $M'^2$  is not sum of the left and right components. However, the mass operator  $M^2$  (unless  $\Omega_K = 1$ ) is not invariant. This implies that the geometries with the radii  $R$  and  $\tilde{R}$  are not equivalent. In other words, geometry is not ambiguous.



For the systems with  $\Omega_K = 1$  (e.g.  $K = 1$ ), we obtain  $M'^2 = M^2$ . Therefore, (19), (21) and (24)–(29) reduce to the usual relations of the usual  $T$ -duality. However, for the systems with  $\Omega_K \neq 1$ , there is  $M'^2 \neq M^2$  and hence the usual  $T$ -duality disappears.

For some of the closed string systems our transformations are the usual  $T$ -duality transformations. These systems have  $\Omega_K = 1$ . However, for some other systems these transformations have feature of the usual  $T$ -duality relations, but are different from them. They have  $\Omega_K \neq 1$ . Thus, we called the whole possibilities (i.e. the usual  $T$ -duality that belongs to  $\Omega_K = 1$ , and modification of it which has  $\Omega_K \neq 1$ ) as the  $T$ -duality-like.

## 5 Conclusions and Summary

We found that the length of each loop of a closed string, wrapped around a compact dimension, up to the order  $\frac{L^2}{L}$  (which is proportional to  $\alpha'/R$ ), is equal to the circle circumference. According to (2), this is consistent with the definition of a compact dimension. Therefore, changing the length of the closed strings modifies the radius of compactification. For example  $K$  closed strings, with positive and negative winding numbers, through the processes splitting and joining produce another system of closed strings. This transition changes the length of the string loops and hence increases the radius of compactification. In other words, initial radius is unstable. The modified radius depends on the winding numbers and the initial radius. Thus, the ratio of the radii is a rational number. If the sum of the initial winding numbers is zero, the initial compactification completely is unstable. In this case after transition the compact dimension is decompactified. This kind of decompactification process may be happened in the first moment of the Universe for some of the dimensions.

We observed that combination of the initial and final (transited) systems by the usual  $T$ -duality leads to inconsistent relations. The inconsistency occurs when the initial system contains more than one closed string. We suggested a solution as in the following. When all closed strings have winding numbers with the same sign, the compactification radius is stable. Thus, the dual radius is given by the usual  $T$ -duality, i.e.  $\alpha'/radius$ . For a given closed string we also considered the exchange of the momentum number with the winding number, similar to the usual  $T$ -duality. When some of the wrapped strings have winding numbers with different signs, we found that the dual radius (i.e.  $\tilde{R}$ ) of the initial compactification (i.e.  $R$ ) depends on all momentum and winding numbers of the initial closed strings. For  $K \geq 2$ , the dual radius  $\tilde{R}$  is less than (for special systems, equal to)  $\alpha'/R$ .

The new form of the dual radius modified the usual duality transformations of the closed string modes. In other words, under the  $T$ -duality-like the compact closed string coordinate does not transform to  $X_L - X_R$ . It has a complicated transformation. In the same way, the closed string mass operator is not invariant. We conclude that the ambiguities in the geometry and topology disappear. In addition, we observed that with an appropriate system of closed strings detecting the distances less than  $\sqrt{\alpha'}$  is possible.

**Acknowledgement** The author would like to thank the referee of “International Journal of Theoretical Physics” for useful comments.

## References

1. Appelquist, T., Chodos, A.: Phys. Rev. D **28**, 772 (1983)
2. Garriga, J., Pujolas, O., Tanaka, T.: Nucl. Phys. B **605**, 192 (2001), [hep-th/0004109](#)
3. Ponton, E., Poppitz, E.: J. High Energy Phys. **0106**, 019 (2001), [hep-ph/0105021](#)

4. Maru, N., Okada, N.: Phys. Rev. D **70**, 025002 (2004), [hep-th/0312148](#)
5. von Gersdorff, G., Hebecker, A.: Nucl.Phys. B **720**, 211–227 (2005), [hep-th/0504002](#)
6. Maru, N., Sakai, N., Uekusa, N.: Radius stabilization by constant boundary superpotentials in warped space. [hep-th/0612071](#) (2006)
7. Giveon, A., Porrati, M., Rabinovici, E.: Phys. Rept. **244**, 77 (1994), [hep-th/9401139](#)
8. Sen, A.: An introduction to duality symmetries in string theory. Prepared for Les Houches Summer School: Session 76: Euro Summer School on Unity of Fundamental Physics: Gravity, Gauge Theory and Strings, Les Houches, France, 30 Jul–31 Aug 2001
9. Seiberg, N.: Emergent spacetime. [hep-th/0601234](#)
10. Aspinwall, P.S., Plesser, M.R.: J. High Energy Phys. **9908**, 001 (1999), [hep-th/9905036](#)
11. Nikolic, H.: Eur. Phys. J. C **50**, 431 (2007), [hep-th/0605250](#)
12. Polchinski, J.: String Theory, vol. I, Chapter 8. Cambridge University Press, Cambridge (1998)
13. Brandenberger, R.H., Vafa, C.: Nucl. Phys. B **316**, 391 (1989)
14. Polchinski, J.: Comm. Math. Phys. **104**, 37 (1986)
15. Bowick, M.J., Wijewardhana, L.C.R.: Phys. Rev. Lett. **54**, 2485 (1985)
16. Frautschi, S.: Phys. Rev. D **3**, 2821 (1971)
17. Atick, J., Witten, E.: IAS preprint IAS-HEP-88/14